NUMERICAL 2-D AND 3-D METHODS FOR COMPUTATION OF INTERNAL UNSTEADY PRESSURE FIELD AND SOUND NEAR FIELD OF FANS

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SUMMARY

Due to a current trend of increasing rotational speed and power, the problem of tone noise and pressure pulsation in centrifugal ventilators becomes a more urgent issue. Often the level of tones determines noise characteristics; mainly these are blade-passing frequencies (BPF). The broadband noise resulting from small-scale turbulent pressure pulsation and secondary flows is an important problem as well. Numerical studies of pressure pulsation in ventilators are undertaken using the method based on a representation of non-stationary motion of a compressible medium as a combination of acoustic and vortex modes.

INTRODUCTION

It is well known that unsteady hydrodynamic interaction of the rotor-outlet flow with the fixed vane cascade or volute tongue of a fan casing is the basic source of noise. As a result of such an interaction acoustical disturbances are formed. They extend upstream and downstream with the speed of sound and reflect from the input and output impedances, creating a specific wave structure inside ventilator casing, depending on Helmholtz number. Near the rotor, in the source of noise oscillations, is a region of pseudo-sound perturbations; they extend with the speed of flow and rapidly attenuate downstream.

Thus inside the fan there are two modes even two zones of perturbations, which differ in the physical nature of oscillations and equations describing their behavior. The first mode is pseudo-sound oscillations caused by unsteady vortex motion of liquid as an incompressible
fluid. The second mode is the acoustic oscillations, which extend throughout entire zone of flow with the speed of sound.

Pressure pulsations depend on quality of hydraulic profiling of ventilator. The amplification of pressure pulsations can happen due to matching of frequencies of oscillations with acoustic resonance frequencies. In ventilators the length of acoustic waves can be comparable to the size of the casing. Therefore the variation of rotation speed, number of rotor blades, the fan installation in the ventilation system can substantially modify amplitudes of pressure pulsations due to the resonance inside the ventilator cavity. Thus there is a necessity of effective numerical methods for a design optimization in regard to the noise and vibration problem.

The fact is that application of common CFD codes for compressible fluid proves to be ineffective for the solution of problem of optimization of design regarding pressure pulsation, especially for the acoustical part of pressure field. Some works were published, in which the methods of prediction of pressure pulsation were developed by a direct computation of non-stationary two-dimensional flow in a centrifugal impeller and volute with solution of averaged Navier-Stokes equations and k-ε model of turbulence [1]. Other approaches [2, 3] used solutions of hydrodynamic equations accompanied with laser anemometric measurements. The most promising approaches proposed 2 – 3 steps computational procedure including solution of Navier-Stokes equation with LES method and consecutive application of an appropriate integral method for resolving far-field noise [4, 5].

For the solution of this problem the method is proposed, which is based on splitting the equations of compressible fluid dynamics into two modes - vortex and acoustic. In this case non-linear equations for unsteady vortex motion of an incompressible liquid are solved with a bigger time step. Wave equation relative to the pressure pulsation taking into account acoustic impedances on the borders of computational domain is solved by an explicit method. As a result a whole processor time for both modes of oscillations has reduced and accuracy of prediction for the acoustical mode is increased. The method is included into 2-D and 3-D software packages.

**ACOUSTICS--VORTEX APPROACH**

**Governing equations**

For prediction of airborne sound in the near field the mathematical model is based on a representation of fluctuating flow velocity field \( V \) as a combination of vortex and acoustic modes [6, 7],

\[
V = U + \nabla \phi = U + V_a
\]  

(1)

Where

- \( U \) - Velocity of transitional and rotational motion of incompressible liquid (vortex mode)

- \( V_a \) - Velocity of pure deformation (acoustic mode)

- \( \phi \) - Acoustic potential

This gives an acoustic-vortex wave equation (2) in terms of static enthalpy oscillation \( \dot{i} \) in the isentropic subsonic flow of compressible fluid.
\[
\frac{1}{a^2} \frac{\partial^2 i}{\partial t^2} - \Delta i = \nabla (\nabla (\frac{1}{2} U^2) - U \times (\nabla \times U))
\]  

(2)

Right hand side of this equation represents the source function, defined from the velocity field of vortex mode flow. It is determined from the solution of unsteady equations of incompressible fluid with appropriate boundary conditions. It must be noted that equation (2) differs in its source term from an equation used in subsonic aeroacoustic applications [8] by the term \(\nabla^2 (\frac{1}{2} U^2)\) representing the kinetic energy of the vortex mode. It also differs in source term from the well-known Lightill equation [9].

There is a simplification in equation (2) with regard to convection of acoustical perturbations by the fluid flow. For higher frequencies it will be more accurate to use the operator \(\frac{\partial}{\partial t} + U \cdot \nabla\) of substantial derivative instead of \(\frac{\partial}{\partial t}\), where \(U\) is the vector of the vortex mode velocity.

For the space scale and the characteristic velocity one takes the impeller radius \(R_2\) and impeller tip velocity \(u_2\). Then the dimensionless quantities of space-vector \((r)\), velocity \((\dot{U})\), time \((t)\) and enthalpy \((i)\) will be as follows:

\[
\tilde{r} = \frac{r}{R_2}; \tilde{U} = \frac{U}{u_2}; \tau = \frac{t}{(2\pi R_2)/(u_2 z_1)}; \tilde{i} = \frac{i}{u_2^2}
\]  

(3)

Where \(z_1\) is the number of main impeller blades. Under assumption of subsonic isentropic flow of a non-viscous fluid one obtains an equation of pressure oscillations (due to acoustical and vortex motion)

\[
\Lambda^2 \frac{\partial^2 h}{\partial \tau^2} - \Delta h = -\Delta g
\]  

(4)

where \(\Lambda\) is Helmholzt similarity criterion of the given problem.

It is simple to show that the parameter \(\Lambda\) is the product of Mach number and Strouhal number and represents a relationship between the impeller tip radius \(R_2\) and the wavelength \(\lambda\) of the main BPF tone

\[
\Lambda = \frac{u_2 z_1}{2\pi a} = f_{bl} R_2 = \frac{u_2}{a} \frac{f_{bl} R_2}{u_2} = M \cdot St = \frac{R_2}{\lambda}
\]  

(5)

where \(f_{bl}\) – main blade passing frequency, \(a\) – speed of sound.

The amplitude of pressure pulsation in an hydraulic machine is at least by an order of magnitude lower than the mean undisturbed pressure, thus for enthalpy oscillations (as a sum of vortex and acoustical perturbations) it is possible to write approximately

\[
h = \tilde{i} - \tilde{i}_0 \approx \frac{(P - P_0)}{\rho_0 u_2^2} = \frac{P'}{\rho_0 u_2^2}
\]  

(6)
Where $P$ is the pressure of compressible fluid, $i_0$, $P_0$ and $\rho_0$ are the mean enthalpy, pressure and density. Similarly for oscillations of the function $g$ we obtain pressure pulsation $(P_v - P_0)$ in “vortex-mode motion”

$$g \approx \frac{(P_v - P_0)}{\rho_0 u_2^2} = \frac{P'_v}{\rho_0 u_2^2}$$

produced by non-stationary vortex motion in the flow of an incompressible fluid – so called "pseudo-sound".

The right hand side of the wave equation (4) is determined from (7, 8) representing unsteady velocity field of the vortex mode (incompressible fluid) flow.

$$- \Delta P_v = \nabla \left[ \nabla \left( \frac{U^2}{2} \right) - \nabla \times (\nabla \times \mathbf{U}) \right]$$

The solution of the equation (4) is divided into two steps - computation of the incompressible flow for the determination of the source function and solution of the inhomogeneous wave equation for the determination of $h$.

The problem of pressure oscillation field determination splits into two or three main steps. The first one is the incompressible liquid flow analysis in the impeller (rotor) to obtain unsteady boundary condition of the vortex mode flow. This boundary condition can be represented in the form of a distribution of rotating parameters’ “attached “ to the impeller exit diameter. The second step is the unsteady vortex mode flow computation into the working cavity of ventilator with consequent determination of the source function, and the third one is solution of the wave equation (4) relatively to pressure oscillations. By using a local complex specific acoustic impedance $Z_k$, the boundary condition for the acoustic mode [10] can be put in the form (9)

$$\frac{\partial (h_k - g_k)}{\partial n} = - \frac{\Lambda k}{Z_k} \frac{\partial (h_k - g_k)}{\partial \tau}$$

where $k$ is a number of BPF harmonic, $n$ – normal direction to the boundary.

**Numerical methods**

For the analysis of the problem, the 2D code Harmony [11--13] and the 3D code FlowVision [14--16] are used.

In 2D analysis the vortex mode flow is computed in 2 steps. In the first step the flow around rotor is defined by the discrete vortex method that applies a “sliding-break-point” conditions on blades. In the second step the unsteady Euler equations are solved for the turbulent flow in the volute casing. Then the source function of the wave equation is defined from the unsteady velocity field of the vortex mode. The computational procedure is the same as for centrifugal ventilators [17--19] and pumps [20--21].
The 3D numerical procedure is based on non-staggered Cartesian grid with adaptive local refinement and a sub-grid geometry resolution method for description of curvilinear complex boundaries [14]. For vortex mode flow, unsteady Navier-Stokes equations are solved with applying \( k-\varepsilon \) turbulence model or using Direct Numerical Simulation (DNS). Iterative procedure goes up to convergence to a periodical solution and subsequent definition of the source function. Initial condition is zero pressure and velocity in entire computational domain. On rigid walls the logarithmic velocity profile is applied as a boundary condition for the turbulent flow. At the outer boundary free-outlet flow condition is used with linear extrapolation of velocity from inner nodes.

Finally wave equation is solved relatively to pressure oscillation using an explicit numerical procedure. Zero pulsatory pressure is an initial condition for solution of the wave equation.

RESULTS OF 2D COMPUTATION

It must be emphasized that that application of acoustic-vortex equation gave a very good agreement with pressure pulsation measurements completed in CETIM [21] with using air pump test rig. The facility (Figure 1) enables to register the entire pulsatory pressure field within the flow part of the pump.

![Figure 1: Air pump unit](image)

Below in Figure 2 one can see a comparison between the experiment and 2D computation for a distribution of amplitude of the first BPF harmonic.

![Figure 2: Air pump model; Amplitude of 1st BPF; left-computation, right – experiment](image)
Such an agreement is reached thanks to accurate simulation of the source function (low-amplitude zone in the volute) and application of the actual impedance (open-end) boundary condition (low-amplitude zone in the outlet).

In figures 3 – 8 are outlined pressure pulsation maps (left) and distributions of amplitude of the first BPF harmonic (right) in different types of centrifugal machines working in air at the Best Efficiency Point (BEP) operation mode. Red and blue zones in the amplitude maps differ by at least 10 dB in the amplitude of the 1\textsuperscript{st} BPF tone. Helmholtz parameter $\Lambda$ representing the relation of the impeller tip radius to the 1\textsuperscript{st} BPF wavelength changes considerably from 0.04 for the air pump (Figure 3) to 0.89 for centrifugal mill (Figure 8). In the air pump the characteristic feature of unsteady pressure in the volute is lower pressure zones linking with blade exit edges and rotating with impeller. They form pseudo-sound oscillations near the impeller exit. For lower $\Lambda$ acoustic perturbations dominate at the volute exit, in conical diffuser. Low-amplitude zone at the pump exit is obtained due to the effect of “open-end-condition”. The impeller of industrial ventilator (Figure 4) has 32 equidistant forward curved blades and rotation speed 960 RPM. Under these conditions Helmholtz number equals 0.37 i.e. the wavelength of BPF pressure pulsation covers approximately one and a half of impeller diameter.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure3.png}
\caption{Air pump; $\Lambda = 0.04$}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure4.png}
\caption{Industrial ventilator; $\Lambda = 0.37$}
\end{figure}

In ventilators (figures 4 – 7) the relative size of vortex perturbations is small because of a relatively large number of blades of centrifugal impeller.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure5.png}
\caption{Blower; $\Lambda = 0.4$}
\end{figure}

In the blower with partial flow inlet (Figure 5) the actual complex impedance boundary condition brings a real resonance behavior of the volute cavity. In the case of high-speed ventilators (figures 6 – 7) the transverse dimension of the volute is comparable with impeller
diameter, transverse mode of acoustical pressure pulsation occurs in the volute. One can see two low-amplitude ("node") zones in the volute in Figure 6 (right) and one node-zone in Figure 7 (right).

![Figure 6: Ventilator model Λ = 0.48](image)

![Figure 7: Motor-Car ventilator Λ = 0.48](image)

In the centrifugal mill (Figure 8) both phenomena – pseudo-sound mode and transverse acoustical mode of oscillations are essential that is found from computational results. It must be noted again that under big Helmholtz number more accurate prediction will be done taking into account convection of acoustical waves with vortex-mode flow.

![Figure 8: Centrifugal mill Λ = 0.89](image)

**RESULTS OF 3D COMPUTATION**

The 3D method is currently validated on centrifugal pumps and lawn mowers. The lawn mower consists of a volute casing and a rotor with two cutting blades. The complex mixed-type flow presents due to the air intake through the gap between the volute and ground [15, 23]. Computational domain is shown in Figure 9.

![Figure 9: Computational domain](image)
The bottom of the cylinder is considered as a rigid wall (ground). Side and top surfaces form the outlet boundary. The volute wall and rotor are considered as absolutely rigid with the logarithmic velocity profile defined on it.

Grid generation procedure produces a rectangular grid with local multilevel adaptation. The mesh is automatically adapted in the volute region resulting in more accurate simulation. The original cell is divided into 8 equally size cells (1st adaptation level). Furthermore the resulting cell can be divided again (2nd adaptation level) and so up to the required level of accuracy.

![Grid with local adaptation (bottom view)](image)

*Figure 10: Grid with local adaptation (bottom view)*

The sub-grid resolution method is used ‘to fit’ the Cartesian grid to the geometrical boundary in order to accurately describe the boundary conditions. It is especially important for the blade that represents a relatively thin and curved surface. The initial “parent” rectangular cell is cut off by the curvilinear surface and the parent cell is disjoined into new volume elements formed by the facet surface and the original grid cell faces. The grid cells also automatically adapted when the rotor passes through them.

![Instantaneous pressure field](image)

*Figure 11: Instantaneous pressure field*

In *Figure 11* one can see unsteady (pseudo-sound) pressure near the ground (a), pulsatory (pseudo-sound and acoustical) pressure field (b) on the volute level and in 2 planes (c). Horizontal plane is near the ground level and vertical plane is in 3 sm from the volute outlet cross-section. Because of a big difference in the amplitude the different scaling is applied (see upper right corner, the pulsatory pressure reduced by a fluid density multiplied by blade tip
velocity squared.). For the horizontal plane the scale is 4 times more than for the vertical one. The pressure field consists from a pseudo-sound zone in the volute casing and diffuse sound field near the lawn mower. Pseudo-sound oscillation is generated directly by rotating blades and the amplitude here equals to the pressure differential on the blade. Near field is created by unsteady fluid motion in vicinity of the casing, outlet air jet and recirculation, emission of sound waves from the exhaust section and from the gap between the casing and the ground. The BPF noise of the lawn mower rapidly attenuates with increase of the distance from the machine.

CONCLUSION

The method for predicting the 3-D sound near field in ventilators and lawn mowers is developed on the basis of the acoustic-vortex representation.

The distribution of the BPF pressure pulsation amplitude within the volute casing of a centrifugal ventilator depends on the Helmholtz parameter $\Lambda$ (relationship between the impeller tip radius and the main BPF wave length) and the volute transverse dimension.

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